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## Comment

# Comment on ‘Fully covariant radiation force on a polarizable particle’

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## Abstract

Recently Pieplow and Henkel (2013 *New J. Phys.* **15** 023027) presented a new fully covariant theory of the Casimir friction force acting on small neutral particle moving parallel to flat surface. We compare results of this theory with results which follow from a fully relativistic theory of friction in plate–plate configurations in the limit when one plate is considered as sufficiently rarefied. We show that there is agreement between these theories.

Keywords: Casimir friction, quantum friction, fluctuating electromagnetic field

## 1. Introduction

All bodies are surrounded by a fluctuating electromagnetic field due to the thermal and quantum fluctuations of the charge and current density inside the bodies. Outside the bodies this fluctuating electromagnetic field exists partly in the form of propagating electromagnetic waves and partly in the form of evanescent waves. The theory of the fluctuating electromagnetic field was developed by Rytov [1–3]. A great variety of phenomena such as Casimir–Lifshitz forces [4], near-field radiative heat transfer [5], and friction forces [6–8] can be described using this theory.

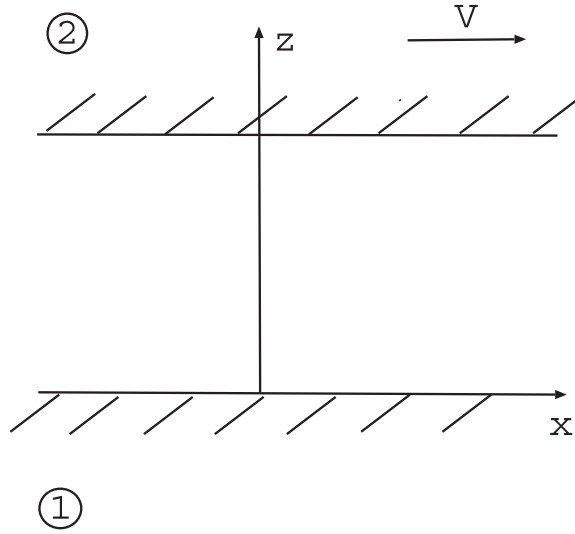


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In [6] we used the dynamical modification of the Lifshitz theory to calculate the friction force between two plane parallel surfaces in parallel relative motion with velocity  $V$ . The calculation of the van der Waals friction is more complicated than of the Casimir–Lifshitz force and the radiative heat transfer because it requires the determination of the electromagnetic field between moving boundaries. The solution can be found by writing the boundary conditions on the surface of each body in the rest reference frame of this body. The relation between the electromagnetic fields in the different reference frames is determined by the Lorentz transformation. In [6] the electromagnetic field in the vacuum gap between the bodies was calculated to linear order in  $V/c$ , which give the contribution to the friction force to order  $(V/c)^2$ . These relativistic corrections were neglected within the non-relativistic theory developed in [6]. The same non-relativistic theory was used in [9] to calculate the frictional drag between quantum wells, and in [10, 11] to calculate the friction force between flat parallel surfaces in normal relative motion. In [12] we presented a rigorous quantum mechanical calculation using the Kubo formula for the friction coefficient. This calculation confirmed the correctness of the approach based on the dynamical modification of the Lifshitz theory, at least to linear order in the sliding velocity  $V$ . For a review of the van der Waals friction see [7].

In [8] we developed a fully relativistic theory of the Casimir–Lifshitz forces and the radiative heat transfer at non-equilibrium conditions, when the interacting bodies are at different temperatures, and they move relative to each other with the arbitrary velocity  $V$ . In comparison with previous calculations [6, 9–11], we did not make any approximation in the Lorentz transformation of the electromagnetic field. This allowed us to determine the field in one reference frame, knowing the same field in another reference frame. Thus, the solution of the electromagnetic problem was exact. Knowing the electromagnetic field we calculated the stress tensor and the Poynting vector which determined the Casimir–Lifshitz forces and the heat transfer, respectively. Taking the limit when one of the bodies is rarefied, it is possible to obtain the Casimir–Lifshitz force and friction, and the radiative heat transfer for a small particle–surface configuration. However, in this approach additional approximations were made which did not allow to make detailed comparison with other theories of friction for the particle–surface configuration in ultra relativistic case.

The problem of friction for a small neutral particle moving parallel to a solid surface (particle–surface configuration) was considered by number of authors (see [7, 13–18], and references therein). At present the interest in this problem is increasing because it is linked to quantum Cherenkov radiation [19–23]. Recently a fully covariant theory of friction in particle–surface configuration was proposed by Pieplow and Henkel (PH) [14] and comparison with results of previous authors was given. The theory presented by PH agrees with relativistic theory proposed by Dedkov and Kyasov (DK) [13]. However, it is well known that the friction between a particle and solid surface, mediated by evanescent electromagnetic waves, can be extracted from friction acting between two plates assuming that one plate is sufficiently rarefied [7]. A fully relativistic theory of friction between two plates in parallel relative motion (plate–plate configuration) was developed in [8]. In the present comment we propose an alternative approach for calculation of the friction force in particle–plate configuration which is calculated from the friction in plate–plate configuration assuming that one plate is sufficiently rarefied. We compare our results with the results of [14] and show that there is agreement between these two theories.



**Figure 1.** Two semi-infinite bodies with plane parallel surfaces separated by a distance  $d$ . The upper solids moves parallel to other with velocity  $V$ .

## 2. Basic results of a fully relativistic theory of friction between two plates at parallel relative motion

We consider two semi-infinite solids having flat parallel surfaces separated by a distance  $d$  and moving with the velocity  $V$  relative to each other, see figure 1. We introduce the two coordinate systems  $K$  and  $K'$  with coordinate axes  $xyz$  and  $x'y'z'$ . In the  $K$  system body 1 is at rest while body 2 is moving with the velocity  $V$  along the  $x$ -axis (the  $xy$  and  $x'y'$  planes are in the surface of body 1,  $x$  and  $x'$ -axes have the same direction, and the  $z$  and  $z'$ -axes point toward body 2). In the  $K'$  system body 2 is at rest while body 1 is moving with velocity  $-V$  along the  $x$ -axis. Since the system is translational invariant in the  $\mathbf{x} = (x, y)$  plane, the electromagnetic field can be represented by the Fourier integrals

$$\mathbf{E}(\mathbf{x}, z, t) = \int_{-\infty}^{\infty} d\omega \int \frac{d^2q}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{x} - i\omega t} \mathbf{E}(\mathbf{q}, \omega, z), \quad (1)$$

$$\mathbf{B}(\mathbf{x}, z, t) = \int_{-\infty}^{\infty} d\omega \int \frac{d^2q}{(2\pi)^2} e^{i\mathbf{q}\cdot\mathbf{x} - i\omega t} \mathbf{B}(\mathbf{q}, \omega, z), \quad (2)$$

where  $\mathbf{E}$  and  $\mathbf{B}$  are the electric and magnetic induction field, respectively, and  $\mathbf{q}$  is the two-dimensional wave vector in  $xy$ -plane. After Fourier transformation it is convenient to decompose the electromagnetic field into  $s$ - and  $p$ -polarized components. For the  $p$ - and  $s$ -polarized electromagnetic waves the electric field  $\mathbf{E}(\mathbf{q}, \omega, z)$  is in plane of incidence, and perpendicular to that plane, respectively. In the vacuum gap between the bodies the electric field  $\mathbf{E}(\mathbf{q}, \omega, z)$ , and the magnetic induction field  $\mathbf{B}(\mathbf{q}, \omega, z)$  can be written in the form

$$\mathbf{E}(\mathbf{q}, \omega, z) = (v_s \hat{n}_s + v_p \hat{n}_p^+) e^{-k_z z} + (w_s \hat{n}_s + w_p \hat{n}_p^-) e^{k_z z}, \quad (3)$$

$$\mathbf{B}(\mathbf{q}, \omega, z) = (v_s \hat{n}_p^+ - v_p \hat{n}_s) e^{-k_z z} + (w_s \hat{n}_p^- - w_p \hat{n}_s) e^{k_z z}, \quad (4)$$

where  $k_z = ((q^2 - (\omega + i0^+/c)^2)^{1/2}$ ,  $\hat{n}_s = [\hat{z} \times \hat{q}] = (-q_y, q_x, 0)/q$ ,  $\hat{n}_p^\pm = [\hat{k}^\pm \times \hat{n}_s] = (\mp q_x i k_z, \mp q_y i k_z, q^2)/(kq)$ ,  $k = \omega/c$ ,  $\hat{k}^\pm = (\mathbf{q} \pm i\hat{z}k_z)/k$ . At the surfaces of the bodies the amplitude of the outgoing electromagnetic wave must be equal to the amplitude of the reflected wave plus the amplitude of the radiated wave. Thus, the boundary conditions for the electromagnetic field at  $z = 0$  in the  $K$ -reference frame can be written in the form

$$v_{p(s)} = R_{1p(s)}(\omega, q)w_{p(s)} + E_{1p(s)}^f(\omega, q), \quad (5)$$

where  $R_{1p(s)}(\omega)$  is the reflection amplitude for surface 1 for the  $p(s)$ -polarized electromagnetic field, and where  $E_{1p(s)}^f(\omega)$  is the amplitude of the fluctuating electric field radiated by body 1 for a  $p(s)$ -polarized wave.

In the  $K'$ -reference frame the electric field can be written in the form

$$\mathbf{E}'(\mathbf{q}', \omega', z) = (v'_s \hat{n}'_s + v'_p \hat{n}'_p) e^{-k_z z} + (w'_s \hat{n}'_s + w'_p \hat{n}'_p) e^{k_z z}, \quad (6)$$

where  $\mathbf{q}' = (q'_x, q'_y, 0)$ ,  $q'_x = (q_x - \beta k)\gamma$ ,  $\omega' = (\omega - Vq_x)\gamma$ ,  $\gamma = 1/\sqrt{1 - \beta^2}$ ,  $\beta = V/c$ ,  $\hat{n}'_s = (-q_y, q'_x, 0)/q'$ ,  $\hat{n}'_p^\pm = (\mp q'_x i k'_z, \mp q'_y i k'_z, q'^2)/(k'q')$ ,

$$q' = \gamma \sqrt{q^2 - 2\beta k q_x + \beta^2 (k^2 - q_y^2)}.$$

The boundary conditions at  $z = d$  in the  $K'$ -reference frame can be written in a form similar to equation (5)

$$w'_{p(s)} = e^{-2k_z d} R_{2p(s)}(\omega', q') v'_{p(s)} + e^{-k_z d} E_{2p(s)}^f(\omega', q'), \quad (7)$$

where  $R_{2p(s)}(\omega)$  is the reflection amplitude for surface 2 for  $p(s)$ -polarized electromagnetic field, and where  $E_{2p(s)}^f(\omega)$  is the amplitude of the fluctuating electric field radiated by body 2 for a  $p(s)$ -polarized wave. A Lorentz transformation for the electric field gives

$$E'_x = E_x, \quad E'_y = (E_y - \beta B_z)\gamma, \quad E'_z = (E_z + \beta B_y)\gamma. \quad (8)$$

Using equations (3), (4), (6) and (8) we get

$$v'_p = \frac{k'\gamma}{kq q'} \left[ -i\beta k_z q_y v_s + (q^2 - \beta k q_x) v_p \right], \quad (9)$$

$$w'_p = \frac{k'\gamma}{kq q'} \left[ i\beta k_z q_y w_s + (q^2 - \beta k q_x) w_p \right], \quad (10)$$

$$v'_s = \frac{k'\gamma}{kq q'} \left[ i\beta k_z q_y v_p + (q^2 - \beta k q_x) v_s \right], \quad (11)$$

$$w'_s = \frac{k'\gamma}{kq q'} \left[ -i\beta k_z q_y w_p + (q^2 - \beta k q_x) w_s \right]. \quad (12)$$

Substituting equations (9)–(12) in equation (7) and using equation (5) we get

$$(q^2 - \beta k q_x) \Delta_{pp} w_p + i\beta k_z q_y \Delta_{sp} w_s$$

$$= e^{-2k_z d} R'_{2p} \left[ (q^2 - \beta k q_x) E_{1p}^f - i\beta k_z q_y E_{1s}^f \right] + \frac{k q q'}{k' \gamma} e^{-k_z d} E_{2p}^f, \quad (13)$$

$$\begin{aligned} & (q^2 - \beta k q_x) \Delta_{ss} w_s - i\beta k_z q_y \Delta_{ps} w_p \\ &= e^{-2k_z d} R'_{2s} \left[ (q^2 - \beta k q_x) E_{1s}^f + i\beta k_z q_y E_{1p}^f \right] + \frac{k q q'}{k' \gamma} e^{-k_z d} E_{2s}^f, \end{aligned} \quad (14)$$

where

$$\Delta_{pp} = 1 - e^{-2k_z d} R_{1p} R'_{2p}, \quad \Delta_{ps} = 1 + e^{-2k_z d} R_{1p} R'_{2s},$$

$\Delta_{ss} = \Delta_{pp} (p \leftrightarrow s)$ ,  $\Delta_{sp} = \Delta_{ps} (p \leftrightarrow s)$ ,  $R'_{2p(s)} = R_{2p(s)} (\omega', q')$ , the symbol  $(p \leftrightarrow s)$  means permutation of the indexes  $p$  and  $s$ . From equations (13), (14) and (5) we get

$$\begin{aligned} w_p = & \left\{ \left[ (q^2 - \beta k q_x)^2 R'_{2p} \Delta_{ss} + \beta^2 k_z^2 q_y^2 R'_{2s} \Delta_{sp} \right] E_{1p}^f e^{-2k_z d} \right. \\ & - i\beta k_z q_y (q^2 - \beta k q_x) (R'_{2p} + R'_{2s}) E_{1s}^f e^{-2k_z d} \\ & \left. + \frac{k q q'}{k' \gamma} \left[ (q^2 - \beta k q_x) \Delta_{ss} E_{2p}^f - i\beta k_z q_y \Delta_{sp} E_{2s}^f \right] e^{-k_z d} \right\} \Delta^{-1}, \end{aligned} \quad (15)$$

$$\begin{aligned} v_p = & \left\{ \left[ (q^2 - \beta k q_x)^2 \Delta_{ss} - \beta^2 k_z^2 q_y^2 \Delta_{sp} \right] E_{1p}^f \right. \\ & - i\beta k_z q_y (q^2 - \beta k q_x) R_{1p} (R'_{2p} + R'_{2s}) e^{-2k_z d} E_{1s}^f \\ & \left. + \frac{k q q'}{k' \gamma} R_{1p} \left[ (q^2 - \beta k q_x) \Delta_{ss} E_{2p}^f - i\beta k_z q_y \Delta_{sp} E_{2s}^f \right] e^{-k_z d} \right\} \Delta^{-1}, \end{aligned} \quad (16)$$

$$\begin{aligned} w_s = & \left\{ \left[ (q^2 - \beta k q_x)^2 R'_{2s} \Delta_{pp} + \beta^2 k_z^2 q_y^2 R'_{2p} \Delta_{ps} \right] E_{1s}^f e^{-2k_z d} \right. \\ & + i\beta k_z q_y (q^2 - \beta k q_x) (R'_{2p} + R'_{2s}) E_{1p}^f e^{2ik_z d} \\ & \left. + \frac{k q q'}{k' \gamma} \left[ (q^2 - \beta k q_x) \Delta_{pp} E_{2s}^f + i\beta k_z q_y \Delta_{ps} E_{2p}^f \right] e^{-k_z d} \right\} \Delta^{-1}, \end{aligned} \quad (17)$$

$$\begin{aligned} v_s = & \left\{ \left[ (q^2 - \beta k q_x)^2 \Delta_{pp} - \beta^2 k_z^2 q_y^2 \Delta_{ps} \right] E_{1s}^f \right. \\ & + i\beta k_z q_y (q^2 - \beta k q_x) R_{1p} (R'_{2p} + R'_{2s}) e^{-2k_z d} E_{1p}^f \\ & \left. + \frac{k q q'}{k' \gamma} R_{1s} \left[ (q^2 - \beta k q_x) \Delta_{pp} E_{2s}^f + i\beta k_z q_y \Delta_{ps} E_{2p}^f \right] e^{-k_z d} \right\} \Delta^{-1}, \end{aligned} \quad (18)$$

where

$$\Delta = (q^2 - \beta k q_x)^2 \Delta_{ss} \Delta_{pp} - \beta^2 k_z^2 q_y^2 \Delta_{ps} \Delta_{sp}.$$

The fundamental characteristic of the fluctuating electromagnetic field is the correlation function, determining the average product of amplitudes  $E_{p(s)}^f(\mathbf{q}, \omega)$ . According to the general theory of the fluctuating electromagnetic field (see for a example [7])

$$\begin{aligned} \langle |E_{p(s)}^f(\mathbf{q}, \omega)|^2 \rangle = & \frac{\hbar \omega^2 i}{2c^2 |k_z|^2} \left( n(\omega) + \frac{1}{2} \right) \left[ (k_z - k_z^*) (1 - |R_{p(s)}|^2) \right. \\ & \left. + (k_z + k_z^*) (R_{p(s)}^* - R_{p(s)}) \right], \end{aligned} \quad (19)$$

where  $\langle \dots \rangle$  denote statistical average over the random field. We note that  $k_z$  is purely imaginary ( $k_z = -ik_z^l$ ) for  $q < \omega/c$  (propagating waves), and real for  $q > \omega/c$  (evanescent waves). The Bose–Einstein factor

$$n(\omega) = \frac{1}{e^{\hbar \omega / k_B T} - 1}.$$

Thus for  $q < \omega/c$  and  $q > \omega/c$  the correlation functions are determined by the first and the second terms in equation (19), respectively.

The force which acts on the surface of body 1 can be calculated from the Maxwell stress tensor  $\sigma_{ij}$ , evaluated at  $z = 0$ :

$$\begin{aligned} \sigma_{ij} = & \frac{1}{4\pi} \int_0^\infty d\omega \int \frac{d^2 q}{(2\pi)^2} \left[ \langle E_i E_j^* \rangle + \langle E_i^* E_j \rangle + \langle B_i B_j^* \rangle + \langle B_i^* B_j \rangle \right. \\ & \left. - \delta_{ij} \left( \langle \mathbf{E} \cdot \mathbf{E}^* \rangle + \langle \mathbf{B} \cdot \mathbf{B}^* \rangle \right) \right]_{z=0}. \end{aligned} \quad (20)$$

Using equations (3), (4) for the  $x$ -component of the force we get

$$\begin{aligned} \sigma_{xz} = & \frac{i}{4\pi} \int_0^\infty d\omega \int \frac{d^2 q}{(2\pi)^2} \frac{q_x}{k^2} \left[ (k_z - k_z^*) \left( \langle |w_p|^2 \rangle + \langle |w_s|^2 \rangle \right) \right. \\ & \left. - \langle |v_p|^2 \rangle - \langle |v_s|^2 \rangle \right] + (k_z + k_z^*) \langle w_p v_p^* + w_s v_s^* - c.c. \rangle. \end{aligned} \quad (21)$$

Substituting equations (15)–(18) for the amplitudes of the electromagnetic field in equation (21), and performing averaging over the fluctuating electromagnetic field with the help of equation (19), we get the  $x$ -component of the force [8]

$$\begin{aligned} F_x = \sigma_{xz} = & \frac{\hbar}{8\pi^3} \int_0^\infty d\omega \int_{q < \omega/c} d^2 q \frac{q_x}{|\Delta|^2} \left[ (q^2 - \beta k q_x)^2 - \beta^2 k_z^2 q_y^2 \right] \\ & \times \left[ (q^2 - \beta k q_x)^2 (1 - |R_{1p}|^2) (1 - |R_{2p}'|^2) |\Delta_{ss}|^2 \right. \\ & \left. - \beta^2 k_z^2 q_y^2 (1 - |R_{1p}|^2) (1 - |R_{2s}'|^2) |\Delta_{sp}|^2 + (p \leftrightarrow s) \right] (n_2(\omega') - n_1(\omega)) \\ & + \frac{\hbar}{2\pi^3} \int_0^\infty d\omega \int_{q > \omega/c} d^2 q \frac{q_x}{|\Delta|^2} \left[ (q^2 - \beta k q_x)^2 - \beta^2 k_z^2 q_y^2 \right] e^{-2k_z d} \end{aligned}$$

$$\begin{aligned} & \times \left[ (q^2 - \beta k q_x)^2 \operatorname{Im} R_{1p} \operatorname{Im} R'_{2p} |\Delta_{ss}|^2 + \beta^2 k_z^2 q_y^2 \operatorname{Im} R_{1p} \operatorname{Im} R'_{2s} |\Delta_{sp}|^2 \right. \\ & \left. + (p \leftrightarrow s) \right] (n_2(\omega') - n_1(\omega)). \end{aligned} \quad (22)$$

The symbol  $(p \leftrightarrow s)$  denotes the terms which can be obtained from the preceding terms by permutation of the indexes  $p$  and  $s$ . The first term in equation (23) represents the contribution to the friction from propagating waves ( $q < \omega/c$ ), and the second term from the evanescent waves ( $q > \omega/c$ ).

### 3. A fully relativistic theory of the Casimir force and friction force, and radiated heat transfer for a small particle moving parallel to a flat surface

According to equation (23) the contribution to the friction force from the evanescent waves is given by

$$\begin{aligned} F_x &= \frac{\hbar}{2\pi^3} \int_0^\infty d\omega \int_{q>\omega/c} d^2q \frac{q_x}{|\Delta|^2} \left[ (q^2 - \beta k q_x)^2 - \beta^2 k_z^2 q_y^2 \right] e^{-2k_z d} \\ & \times \left[ \operatorname{Im} R_{1p} \operatorname{Im} \Delta_p + (p \leftrightarrow s) \right] (n_2(\omega') - n_1(\omega)), \end{aligned} \quad (23)$$

where

$$\Delta_p = (q^2 - \beta k q_x)^2 R'_{2p} |\Delta_{ss}|^2 + \beta^2 k_z^2 q_y^2 R'_{2s} |\Delta_{sp}|^2,$$

$\Delta_s = \Delta_p (p \leftrightarrow s)$ . If in equation (23) one neglects the terms of the order  $\beta^2$  then the contributions from waves with  $p$ - and  $s$ -polarization will be separated. In this case equation (23) is reduced to the formula obtained in [6]

$$\begin{aligned} F_x &= \frac{\hbar}{2\pi^3} \int_0^\infty d\omega \int_{q>\omega/c} d^2q q_x e^{-2k_z d} \\ & \times \left( \frac{\operatorname{Im} R_{1p} \operatorname{Im} R'_{2p}}{|\Delta_{pp}|^2} + \frac{\operatorname{Im} R_{1s} \operatorname{Im} R'_{2s}}{|\Delta_{ss}|^2} \right) (n_2(\omega') - n_1(\omega)). \end{aligned} \quad (24)$$

Thus, to the order  $\beta^2$  the mixing of waves with different polarization can be neglected, what agrees with the results obtained in [6]. At  $T = 0$  K the propagating waves do not contribute to friction but the contribution from evanescent waves is not equal to zero. Taking into account that  $n(-\omega) = -1 - n(\omega)$  from equation (23) we get the friction mediated by the evanescent electromagnetic waves at zero temperature (in literature this type of friction is denoted as quantum friction [24])

$$\begin{aligned} F_x &= -\frac{\hbar}{\pi^3} \int_0^\infty dq_y \int_0^\infty dq_x \int_0^{q_x^V} d\omega \frac{q_x}{|\Delta|^2} \left[ (q^2 - \beta k q_x)^2 - \beta^2 k_z^2 q_y^2 \right] e^{-2k_z d} \\ & \times \left[ \operatorname{Im} R_{1p} \operatorname{Im} \Delta_p + \operatorname{Im} R_{1s} \operatorname{Im} \Delta_s \right]. \end{aligned} \quad (25)$$

If in equation (25) one neglects the terms of the order  $\beta^2$  then the contributions from waves with  $p$ - and  $s$ -polarization will be separated. In this case equation (25) is reduced to the formula



obtained by Pendry for  $p$ -polarized waves in the non-retarded limit [24]

$$F_x = -\frac{\hbar}{\pi^3} \int_0^\infty dq_y \int_0^\infty dq_x \int_0^{q_x^V} d\omega q_x \left( \frac{\text{Im } R_{1p} \text{Im } R'_{2p}}{|\Delta_{pp}|^2} + \frac{\text{Im } R_{1s} \text{Im } R'_{2s}}{|\Delta_{ss}|^2} \right) e^{-2k_z d}. \quad (26)$$

The friction force acting on a small particle moving in parallel to a flat surface can be obtained from the friction between two semi-infinite bodies in the limit when one of the bodies is sufficiently rarefied. We will assume that the rarefied body consists of small particles which have electric dipole moments. We assume that the dielectric permittivity of this body, say body 2, is close to the unity, i.e.  $\epsilon_2 - 1 \rightarrow 4\pi n\alpha \ll 1$ , where  $n$  is the concentration of particles in body 2 in the co-moving reference frame  $K'$ ,  $\alpha$  is their electric polarizability. To linear order in the concentration  $n$  the reflection amplitudes are

$$R'_{2p} = \frac{\epsilon'_2 k_z - \sqrt{k_z^2 - (\epsilon'_2 - 1)k'^2}}{\epsilon'_2 k_z + \sqrt{k_z^2 - (\epsilon'_2 - 1)k'^2}} \approx \frac{\epsilon'_2 - 1}{4} \frac{q'^2 + k_z^2}{k_z^2} = n\pi \frac{q'^2 + k_z^2}{k_z^2} \alpha',$$

$$R'_{2s} = \frac{k_z - \sqrt{k_z^2 - (\epsilon'_2 - 1)k'^2}}{k_z + \sqrt{k_z^2 - (\epsilon'_2 - 1)k'^2}} \approx \frac{\epsilon'_2 - 1}{4} \frac{q'^2 - k_z^2}{k_z^2} = n\pi \frac{q'^2 - k_z^2}{k_z^2} \alpha'.$$

To linear order in the concentration  $n$  the functions  $\Delta_{pp}$ ,  $\Delta_{ss}$ ,  $\Delta_{sp}$  and  $\Delta_{ps}$  should be calculated at  $n = 0$ . Using that  $\Delta_{pp} = \Delta_{ss} = \Delta_{sp} = \Delta_{ps} = 1$  for  $n = 0$ , we get

$$\Delta = (q^2 - \beta k q_x)^2 - \beta^2 k_z^2 q_y^2 = \frac{(qq')^2}{\gamma^2},$$

$$\Delta_p = \left\{ q'^2 \left[ (q^2 - \beta k q_x)^2 + \beta^2 k_z^2 q_y^2 \right] + k_z^2 \left[ (q^2 - \beta k q_x)^2 - \beta^2 k_z^2 q_y^2 \right] \right\} \frac{\pi n \alpha'}{k_z^2}$$

$$= q'^2 \left\{ q^2 \left[ k_z^2 + (k - \beta q_x)^2 \right] + k_z^2 \left[ q^2 - 2\beta^2 q_x^2 \right] \right\} \frac{\pi n \alpha'}{k_z^2}$$

$$= q'^2 \left\{ q^2 (k - \beta q_x)^2 + 2k_z^2 (q^2 - \beta^2 q_x^2) \right\} \frac{\pi n \alpha'}{k_z^2},$$

$$\Delta_s = \left\{ q'^2 \left[ (q^2 - \beta k q_x)^2 + \beta^2 k_z^2 q_y^2 \right] - k_z^2 \left[ (q^2 - \beta k q_x)^2 - \beta^2 k_z^2 q_y^2 \right] \right\} \frac{\pi n \alpha'}{k_z^2}$$

$$= q'^2 \left\{ q^2 \left[ k_z^2 + (k - \beta q_x)^2 \right] - k_z^2 \left[ q^2 - 2\beta^2 q_y^2 \right] \right\} \frac{\pi n \alpha'}{k_z^2}$$

$$= q'^2 \left\{ q^2 (k - \beta q_x)^2 + 2k_z^2 \beta^2 q_y^2 \right\} \frac{\pi n \alpha'}{k_z^2},$$

where  $\alpha' = \alpha(\omega')$ .

The friction force acting on a particle moving parallel to a plane surface can be obtained as the ratio between the change of the frictional shear stress between two surfaces after displacement of body 2 by small distance  $dz$ , and the number of the particles in a slab with thickness  $dz$ :

$$f_x^{\text{part}} = \left. \frac{dF_x(z)}{n' dz} \right|_{z=d} = \frac{\hbar}{\gamma \pi^2} \int_0^\infty d\omega \int_{q>\omega/c} d^2 q \frac{q_x}{k_z} e^{-2k_z d} \times \left[ \text{Im } R_{1p}(\omega) \phi_p + \text{Im } R_{1s}(\omega) \phi_s \right] \text{Im } \alpha(\omega') (n_2(\omega') - n_1(\omega)), \quad (27)$$

where  $n' = \gamma n$  is the concentration of particles in body 2 in the reference frame  $K$

$$\phi_p = (\omega'/c)^2 + 2\gamma^2 (q^2 - \beta^2 q_x^2) \frac{k_z^2}{q^2}$$

$$\phi_s = (\omega'/c)^2 + 2\gamma^2 \beta^2 q_y^2 \frac{k_z^2}{q^2}.$$

At  $T_2 = T_1 = 0$  K we get

$$f_x^{\text{part}} = -\frac{\hbar}{\gamma \pi^2} \int_{-\infty}^\infty dq_y \int_0^\infty dq_x \int_0^{q_x v} d\omega \frac{q_x}{k_z} e^{-2k_z d} \times \left[ \text{Im } R_{1p}(\omega) \phi_p + \text{Im } R_{1s}(\omega) \phi_s \right] \text{Im } \alpha(\omega'). \quad (28)$$

For  $\beta^2 \ll 1$  and  $q \gg \omega/c$ , equation (27) is reduced to the result of non-relativistic theory [7]

$$f_x^{\text{part}} = \frac{2\hbar}{\pi^2} \int_0^\infty d\omega \int_{q>\omega/c} d^2 q q_x q e^{-2qd} \text{Im } R_{1p} \text{Im } \alpha(\omega - q_x v) (n_2(\omega') - n_1(\omega)). \quad (29)$$

The heat absorbed by the body 1 in the  $K$  system in the plate–plate configuration is determined by the expression which is very similar to the expression for the friction force (23) [8]

$$P_1 = \frac{\hbar}{2\pi^3} \int_0^\infty d\omega \int_{q>\omega/c} d^2 q \frac{\omega}{|\Delta|^2} \left[ (q^2 - \beta k q_x)^2 - \beta^2 k_z^2 q_y^2 \right] e^{-2k_z d} \times \left[ \text{Im } R_{1p} \text{Im } \Delta_p + \text{Im } R_{1s} \text{Im } \Delta_s \right] (n_2(\omega') - n_1(\omega)). \quad (30)$$

Using result obtained for the friction in the particle–surface configuration from (30) and (27) we get the heat absorbed by plate in the  $K$  system in the particle–plate configuration

$$P_1^{\text{part}} = \frac{\hbar}{\gamma \pi^2} \int_0^\infty d\omega \int_{q>\omega/c} d^2 q \frac{\omega}{k_z} e^{-2k_z d} \times \left[ \text{Im } R_{1p}(\omega) \phi_p + \text{Im } R_{1s}(\omega) \phi_s \right] \text{Im } \alpha(\omega') (n_2(\omega') - n_1(\omega)). \quad (31)$$

The heat absorbed by a particle in the  $K'$  system ( $P'_2$ ) can be obtained from the relation

$$f_x V = P_1 + \frac{P'_2}{\gamma}, \quad (32)$$

which follows from the Lorentz transformation of the Poynting vector. From (32) we get

$$P'_2 = \frac{\hbar}{\gamma\pi^2} \int_0^\infty d\omega \int_{q>\omega/c} d^2q \frac{\omega}{k_z} e^{-2k_z d} \times \left[ \text{Im } R_{1p}(\omega') \phi_p + \text{Im } R_{1s}(\omega') \phi_s \right] \text{Im } \alpha(\omega) (n_1(\omega') - n_2(\omega)), \quad (33)$$

where we transformed variables  $\omega, q_x$  in the integrands (27) and (33) to  $\omega', q'_x$  using the fact that the Lorentz transformation has unit Jacobian. After such changing we denoted ‘dummi’ variable  $\omega', q'_x$  as  $\omega, q_x$ .

The Casimir force between two moving plates mediated by the evanescent waves is given by [8]

$$F_z = \frac{\hbar}{4\pi^3} \text{Im} \int_0^\infty d\omega \int_{q>\omega/c} d^2q \frac{k_z}{\Delta} e^{-2k_z d} [R_{1p}\Delta_{1p} + R_{1s}\Delta_{1s}] [1 + n_1(\omega) + n_2(\omega')] + \frac{\hbar}{4\pi^3} \int_0^\infty d\omega \int_{q>\omega/c} d^2q \frac{k_z}{|\Delta|^2} \left[ (q^2 - \beta k q_x)^2 - \beta^2 k_z^2 q_y^2 \right] e^{-2k_z d} \times \left\{ \text{Im } R_{1p} \text{Re } \Delta_p - \text{Re } R_{1p} \text{Im } \Delta_p + (p \leftrightarrow s) \right\} (n_1(\omega) - n_2(\omega')), \quad (34)$$

where

$$\Delta_{1p} = (q^2 - \beta k q_x)^2 R'_{2p} \Delta_{ss} + \beta^2 k_z^2 q_y^2 R'_{2s} \Delta_{sp},$$

$\Delta_{1s} = \Delta_{1p}(p \leftrightarrow s)$ . In the limit  $n \rightarrow 0$ :  $\Delta_{1p(s)} = \Delta_{p(s)}$ . After similar calculations as above for the Casimir force acting on a small particle moving parallel to a flat surface we get

$$F_z^{\text{part}} = \frac{\hbar}{2\gamma\pi^2} \int_0^\infty d\omega \int_{q>\omega/c} d^2q e^{-2k_z d} \left\{ \left[ \phi_p \text{Im } R_{1p} + \phi_s \text{Im } R_{1s} \right] \text{Re } \alpha' \coth \left( \frac{\hbar\omega}{k_B T_1} \right) + \left[ \phi_p \text{Re } R_{1p} + \phi_s \text{Re } R_{1s} \right] \text{Im } \alpha' \coth \left( \frac{\hbar\omega'}{k_B T_2} \right) \right\}. \quad (35)$$

#### 4. Discussion and summary

Recently Pieplow and Henkel [14] presented a fully covariant theory of the Casimir force and the friction force acting on a small neutral particle moving parallel to a flat surface. This theory is in agreement with the relativistic theory presented by Dedkov and Kyasov [13]. In this comment we have shown that the results of PH and DK for contribution to friction from the evanescent waves in the particle–plate configuration are determined by the first derivative of the friction force on the particle concentration in the plate–plate configuration assuming that one of the plate is sufficiently rarefied. However, the inverse procedure is not possible. It is not possible to recover the whole function knowing only its first derivative. The contribution to

friction from the propagating waves is more delicate. To make a comparison between the contributions to friction from propagating waves in the particle–plate configuration and the plate–plate configuration it is necessary to consider a slab with a finite thickness and to calculate the friction force acting on the both side of a slab. In contrast to the evanescent waves, which do not contribute to the friction force acting on the back side of a slab, the propagating waves contribute to the friction force acting on the both side of the slab. However, for large velocities (for example, above the Cherenkov threshold velocity) the friction is dominated by quantum friction determined by the evanescent waves. Classically, Cherenkov radiation is emitted when a charged particle is moving in a medium. Recently, it was shown that a quantum analog of the Cherenkov effect appears when two planar objects are in relative motion beyond a threshold velocity set by the speed of light inside the medium [19, 20]; this effect was originally discovered by Frank and Ginzburg [25]. A similar effect exists in the case of driving of current density in a graphene sheet located on the surface of polar dielectric (like SiO<sub>2</sub>) or nearby of other graphene sheet [26, 27]. A strong enhancement of quantum friction occurs when a drift velocity of electrons in the graphene sheet exceeds a threshold velocity set by the Fermi velocity of the electrons in graphene. The analog of quantum Cherenkov radiation is related to emission of the surface phonon polaritons and electron–hole pairs. Another situation is when a particle is moving through a small channel drilled into a dielectric or parallel to a surface [7, 13–18, 23]. A recent work [21, 22], also explores quantum friction beyond Cherenkov velocity. There are discrepancies between the results of this work and a large number of papers devoted to the study of quantum friction, which need to be further investigated. However, detailed discussion of these discrepancies is outside the scope of the present article. We only draw the attention of the readers on the contradictions between these papers.

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